

Absence of lattice strain anomalies at the electronic topological transition in zinc at high pressure

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High pressure structural distortions of the hexagonal close packed (hcp) element zinc have been a subject of controversy. Earlier experimental results and theory showed a large anomaly in lattice strain with compression in zinc at about 10 GPa which was explained theoretically by a change in Fermi surface topology. Later hydrostatic experiments showed no such anomaly, resulting in a discrepancy between theory and experiment. We have computed the compression and lattice strain of hcp zinc over a wide range of compressions using the linearized augmented plane wave (LAPW) method paying special attention to k-point convergence. We find that the behavior of the lattice strain is strongly dependent on k-point sampling, and with large k-point sets the previously computed anomaly in lattice parameters under compression disappears, in agreement with recent experiments.

I. INTRODUCTION

Zinc and cadmium are unique among the hexagonal close-packed (hcp) transition metals in that the axial ratio ($c/a = 1.856$ for zinc and 1.886 for cadmium) is far from the ideal value defined by hard sphere packing ($c/a = \sqrt{8/3} = 1.633$). Upon compression, the axial ratio decreases towards the ideal value. Lynch and Drickamer¹ first observed that the decrease in c/a with increasing pressure was not smooth; subsequent experiments yielded inconsistent results on the nature of this anomaly^{2–7}. Takemura confirmed the anomaly using a methanol-ethanol-water mixture^{4,7} (MEWM) as a pressure medium in diamond anvil cell experiments: he observed the a -axis expanding over a small range of compression, yielding a rapid decrease of the axial ratio c/a . *Ab initio* computations found similar behavior^{8–10} and provided an explanation for the anomaly by means of changes in the Fermi surface topology under compression^{9,11}. Takemura recently repeated his experiments but using helium as pressure medium¹² which is more nearly hydrostatic than MEWM, but found that both axes compressed monotonically with no anomaly in c/a , contrary to his earlier experiments and theory.

The most recent experimental results call previous theoretical studies^{8–10} into question. All previous theoretical studies show the anomaly in the axial ratio whether the local density approximation (LDA)^{8,9} or the generalized gradient approximation (GGA)¹⁰ to the exchange correlation potential is used. The anomaly has been connected to changes in the electronic structure^{9,11}: Fast *et al.*⁹ observe one electronic topological transition (ETT) at the high symmetry point K on the Brillouin zone boundary forming a ellipsoidal piece (needle) in the Fermi surface. Novikov *et al.*¹¹ see at least one additional ETT at approximately the same compression, also at K, where disconnected pieces form a three-leg structure of the Fermi surface along the K-M directions upon compression. Depending on c/a with compression Novikov *et al.*¹¹ propose one more ETT at L (butterfly) reconciling previous contradictory results from first principles calculations¹³. All previous computations were performed with typical Brillouin zone sampling (no more than 1000 k-points in the irreducible wedge of the Brillouin zone).

In an attempt to understand the discrepancy of the hydrostatic experiments¹² and previous computational results^{9,10} we calculate the equation of state, lattice constants, and electronic structure of zinc over a wide compression range from first principles paying particular attention to the convergence of the calculations with respect to reciprocal space integration. In section II we introduce the method used and elaborate the computational details of our first principles calculations. Section III focuses on our results for the equation of state, lattice constants, and electronic structure. We compare our results to experiments at ambient conditions and high pressure and to previous theoretical work. Discussion and conclusions follow.

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II. METHOD

We investigate the energetics of hcp zinc using the full-potential linearized-augmented plane-wave method (LAPW)¹⁴ with GGA¹⁵. Core states are treated self-consistently using the full Dirac equation for the spherical part of the potential, while valence states are treated in a semirelativistic approximation neglecting spin-orbit coupling. $3s$, $3p$, $3d$, $4s$, and $4p$ states are treated as valence electrons. The muffin-tin radius R_{MT} is 2.0 Bohr over the whole compression range considered.

We perform calculations at three sets of Brillouin zone sampling, 24x24x24, 32x32x32, and 48x48x48 special k-points¹⁶, yielding 732, 1632, and 5208 k-points in the irreducible wedge of the Brillouin zone for the hcp lattice, respectively. The lowest k-point sampling is comparable to the previous GGA study¹⁰ while the latter two are much denser than any previously published results. The size of the basis is set by $R_{MT}K_{max} = 9.0$, where K_{max} is the largest reciprocal space vector. We use Fermi broadening with an electronic temperature of 5 mRy. For the densest k-point mesh we also perform calculations without electronic broadening for a selected subset of volumes and do not see any significant change in our results: equilibrium axial ratios remain within ± 0.005 , the uncertainty of our results.

We carry out total energy calculations over a wide range of unit cell volumes. At each volume we perform calculations for several different values of the axial ratio and find the equilibrium structure by fitting the results to a quadratic. The equation of state is obtained by describing the energy-volume curve with a third order expansion in Eulerian finite strain¹⁷. We consider unit cell volumes ranging from 60-110 Bohr³ for 24x24x24 k-points and focus on the range in which the anomaly in c/a occurs (90-102.5 Bohr³) for 32x32x32 and 48x48x48 k-point meshes.

III. RESULTS

Equation of State

A comparison of the pressure-volume relation between our results and static experiments^{7,12} (Fig. 1 and Table I) shows good agreement at low pressure. At higher pressure theory differs significantly from the results of the MEWM diamond anvil cell experiments⁷; this is consistent with previous theoretical results¹⁰.

To investigate whether non-hydrostaticity may be responsible for the discrepancy we also compare to the results of shock wave experiments¹⁸ where hydrostaticity is readily achieved¹⁹ (Fig. 1 and Table I). The Hugoniot is reduced to a 0 K isotherm by solving the Rankin-Hugoniot equation²⁰. We estimate the thermal pressure ($P_{th} = \gamma E_{th}/V$) along the Hugoniot, with γ the Grüneisen parameter and E_{th} the thermal energy. We approximate the thermal energy by the Dulong Petit law ($C_V^{lat} = 3R$); the electronic contribution to the thermal pressure is negligible (the temperature along the Hugoniot is less than 2000 K). We assume the Grüneisen parameter is proportional to compression ($\gamma = \gamma_0 V/V_0$) with γ_0 its zero pressure value evaluated from the thermodynamic definition ($\gamma = \alpha K_T/C_V \rho$), where the thermal expansivity α , the isothermal bulk modulus K_T , specific heat C_V and density ρ at zero pressure are taken from the literature²¹.

The reduced Hugoniot agrees with our GGA results much better than the static experiments; differences in volume are less than 1.5%. The large discrepancy between the static and shock wave experiments indicates that the MEWM experiments⁷ may be biased by non-hydrostaticity.

Lattice Constants

Total energy as a function of axial ratio for the 24x24x24 k-point mesh shows an unusually large scatter about the quadratic fit in c/a (Fig. 2). With increasing number of k-points the scatter decreases and the minimum becomes better defined. In contrast to the previous GGA results¹⁰ we do not see multiple minima in c/a for any volume and find the axial ratio reliably resolved to within ± 0.005 , within the symbol size in Fig. 1. The curvature of energy as a function of c/a varies considerably for the different k-point meshes, showing that elastic constants will also be strongly dependent on k-point sampling, as the shear elastic constant (C_S) is related to this strain²².

The development of the axial ratio c/a with compression differs for the three sets of computations considerably (Fig. 1). For 24x24x24 k-points we see an anomaly similar to that in the MEWM experiments^{4,7}: after an initial linear decrease in the axial ratio (102.5-95 Bohr³) the slope in c/a steepens (95-90 Bohr³) before decreasing again at higher pressures. The dependence of c/a on compression for k-point meshes of 32x32x32 and 48x48x48 is much smoother; the anomaly in c/a has disappeared. The difference between experiment and theory is less than 4% in c/a which is typical of all electron calculations²³. At higher compression ($V < 70$ Bohr³) the theoretical value smoothly approaches 1.61; the MEWM experiments converge to 1.59.

The nature of the anomaly is revealed by considering the lattice constants separately (Fig. 3). The c -axis compresses monotonically with decreasing volume in all computations and experiments considered. Theory overpredicts c by less than 2%, and there is little difference in c for the two denser k-point meshes. An expansion of the a -axis for the 24x24x24 k-point calculations and the MEWM experiments cause the anomaly in c/a (Fig. 3). For the two denser k-point meshes a compresses monotonically; for volumes smaller than $V = 95$ Bohr³ a follows a linear trend with the same slope as the helium experiments. For volumes greater than $V=95$ Bohr³, a is less compressible than it is at higher pressure. For the larger k-point samplings the calculations underestimate a by less than 1%, while with 24x24x24 k-point the maximum difference is approximately 1.5%.

To illustrate this point further we evaluate the linear compressibility for the two axes $k_x = -(1/x)(\partial x/\partial P)$ (with $x = a, c$) for our results and the static experiments^{7,12} using central differences (Fig. 4). Our results for 24x24x24 k-points show an anomaly in k_a similar in character and magnitude to that found in the MEWM experiments. For denser k-point sampling the anomaly in k_a is shifted towards lower pressure and its magnitude decreases with increasing number of k-points. For k_c an anomaly exists as well for both the MEWM experiments⁷ and the calculations with the smallest k-point mesh, it is however less pronounced than for k_a and is absent from the results for the two denser k-point meshes.

Electronic Structure

The band structure of zinc under compression changes considerably from $V = 102.5$ Bohr³ to $V = 95$ Bohr³ (Fig. 5). The electronic structure is in excellent agreement with the previous GGA results¹⁰. The major change in band structure occurs at the high symmetry point K on the Brillouin zone boundary where three bands (K_7, K_8, K_9 ²⁴) cross the Fermi energy under compression, changing the topology of the Fermi surface. From the band structure we see the needle around K and also the connection of the three-leg piece at K. Focusing on the development of the band structure at K we consider the eigenvalues of the K_7 , K_8 , and K_9 states (Fig. 6). For the 24x24x24 k-point calculations these bands show a quadratic volume dependence and cross the Fermi energy at $V=97.5$ Bohr³ (K_7 and K_8) and $V=97$ Bohr³ (K_9). For the two denser k-point meshes the eigenvalues depend linearly on volume and the crossing points are indistinguishable for 32x32x32 and 48x48x48 k-points. The crossings occur at slightly higher volume than for the 24x24x24 k-point mesh ($V=98$ and 97.5 Bohr³ for K_7 , K_8 , and K_9 , respectively), but the difference is small compared with the effect of k-point sampling on the lattice parameters.

IV. DISCUSSION

The ETT discussed in the last section has previously been invoked as an explanation for the anomaly in a -axis compressibilities^{9,11}. In contrast to these studies we find that the occurrence of the ETT is independent of the calculated anomaly in a -axis compressibility, as the location of abnormal compression of a shifts with increasing k-point sampling towards higher volumes (Figs. 3 and 4) while the ETT always occurs at approximately the same volume (Fig. 6). The anomaly in a -axis compression seen in previous calculations appears to be a consequence of insufficient k-point sampling. The results presented here for a (Fig. 3) and linear compressibility k_a (Fig. 4) suggest that even for the densest k-point mesh we use (48x48x48) the lattice parameters are not converged.

The discrepancy between the MEWM and helium experiments can be attributed to freezing of the MEWM pressure medium which is known to occur at about 10 GPa²⁵. Freezing substantially increases the non-hydrostatic component of stress as recognized previously in high pressure experiments on forsterite (Mg_2SiO_4)²⁶. At room temperature helium also freezes within the pressure range of the experiment (11.5 GPa)²⁷ but remains soft enough to maintain hydrostaticity²⁶. Recent neutron inelastic-scattering experiments under compression^{28,29} show no softening or anomaly in the phonon frequency, supporting the monotonic compression of both axes as seen in our dense k-point calculations and the helium experiments.

In retrospect it occurs as a fortuitous (or unfortunate) coincidence that for *typical* computational parameters comparable behavior in linear compressibilities is found in first principles electronic structure calculations and for experiments with non-hydrostatic conditions, despite the fundamentally different underlying physical problem. Following the same notion the observation of anomalies in the axial ratio under compression for cadmium in experiments⁷ and theory using both LDA³⁰ and GGA¹⁰ may also be an artifact due to non-hydrostatic conditions in the experiments and insufficient convergence with respect to computational parameters as well.

The ETT, however, might have important effects on higher order physical properties such as elasticity. For tantalum a similar change in electronic structure as for zinc has been found under compression which has little effect on the equation of state³¹ but appears in the elastic constants³².

V. CONCLUSIONS

Using the first-principles LAPW method with GGA we calculate the equation of state, structural parameters, and electronic structure of zinc over a wide compression range. We perform calculations for three different k-point samplings of the first Brillouin zone (24x24x24, 32x32x32, and 48x48x48 k-points) and find lattice parameters, in particular the *a*-axis, strongly dependent on the number of k-points, while little or no effect can be seen on the equation of state and band structure. For lattice constants we find that a previously observed anomaly in *a*-axis compressibility shifts to lower pressure and decreases in amplitude as we increase k-point sampling from 24x24x24 to 48x48x48. This anomaly is not coupled to a change in electronic band structure as has been proposed before; we observe the ETT occurring at approximately the same volume for all sets of computational parameters.

The disappearance of the anomaly in lattice constants for our results is in agreement with recent static experiments using helium as a pressure medium. The remaining anomaly in *a*-axis compressibility indicates that structural parameters are not fully converged even for the prohibitively large k-point sampling we perform.

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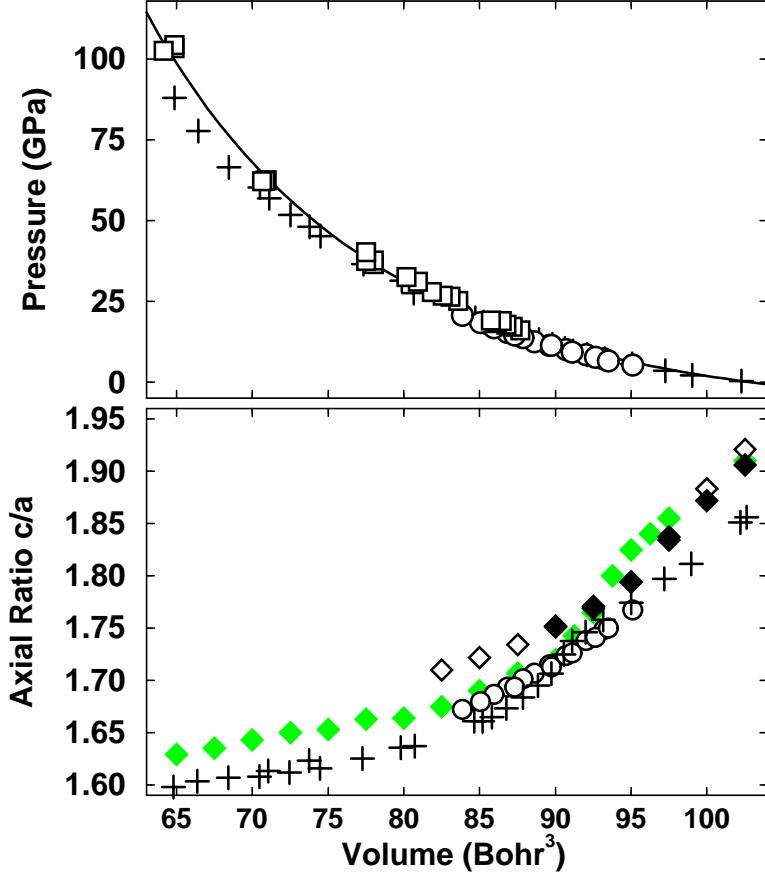


FIG. 1. Axial ratio c/a and equation of state for zinc from our calculations and experiment. The lower panel compares our results for c/a (gray diamonds 24x24x24 k-points, open diamonds 32x32x32 k-points, and filled diamonds 48x48x48 k-points) to static experiments using methanol ethanol water mixture (Ref. 7, pluses) and helium (Ref. 12, circles). The equation of state for zinc is shown in the upper panel for our calculations (line) and the same two set of diamond anvil cell experiments (same symbols as above) The open squares show shock wave experiments (Ref. 18) reduced to a 0 K isotherm.

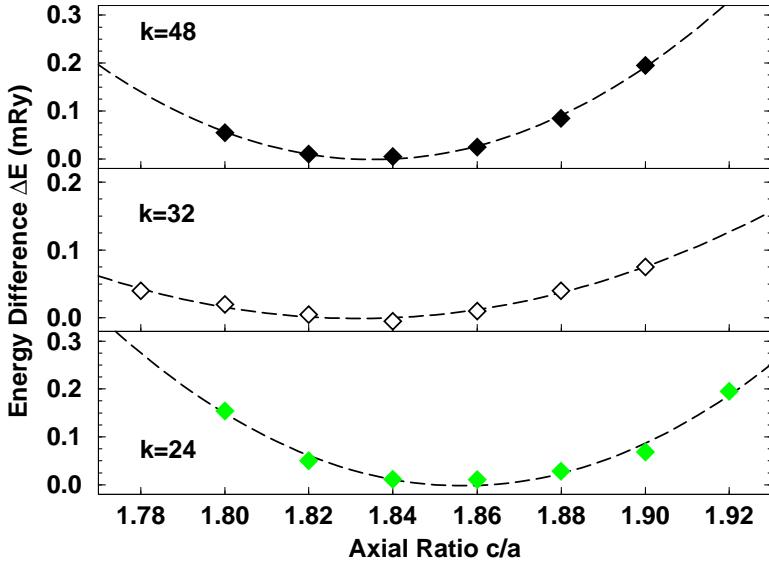


FIG. 2. Relative energies as a function of the axial ratio c/a for $V = 97.5$ Bohr 3 . The lower, middle, and upper panel shows results for 24x24x24, 32x32x32, and 48x48x48 k-points, respectively. The lines show quadratic fits in c/a to the results.

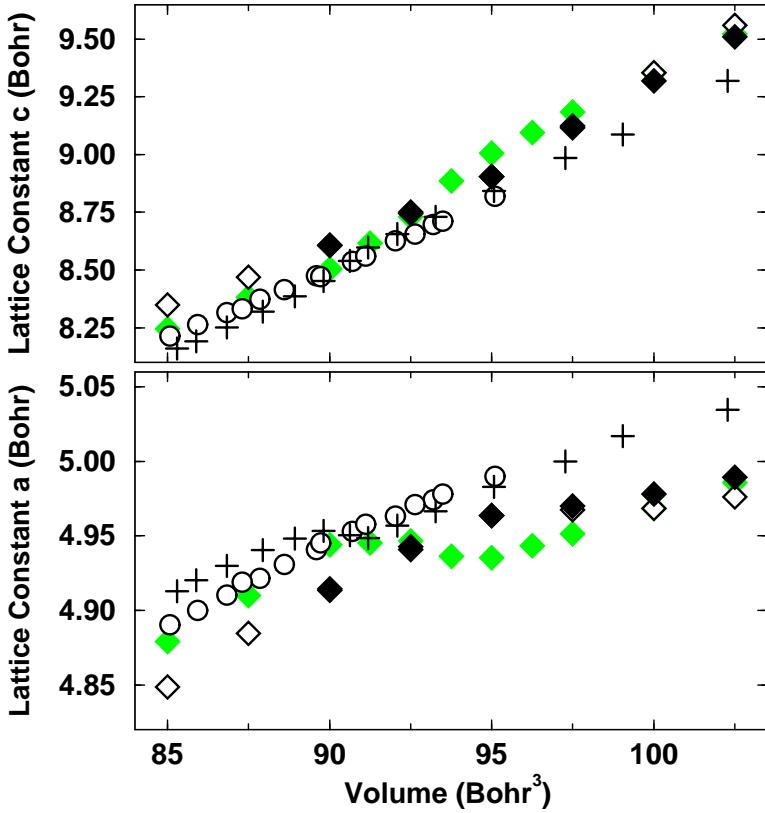


FIG. 3. Compression of the two axes a and c in the hexagonal cell for zinc over the compression range $V/V_0 = 0.80 - 1.0$. In the upper panel we compare our results (diamonds) for c (gray 24x24x24, open 32x32x32, and filled 48x48x48 k-points) with the static experiments by Takemura using methanol-ethanol-water mixture (pluses, Ref. 7) and helium (open circles, Ref. 12) as a pressure medium. For a in the lower panel the same symbols are used. Note the approximately fivefold difference in range of axes compressibilities for c and a .

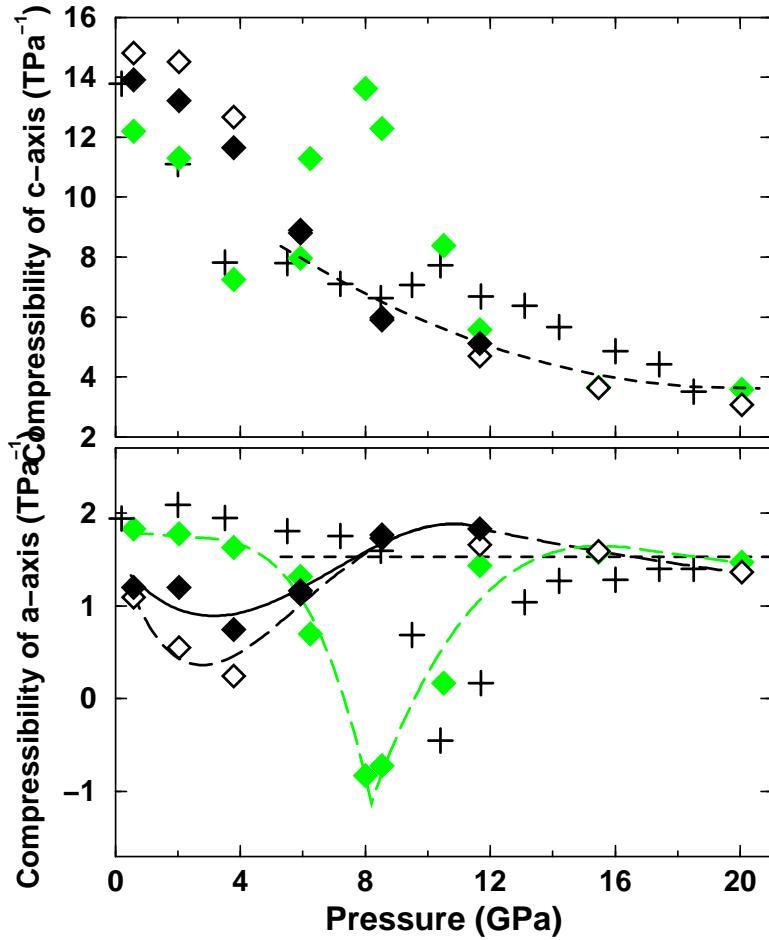


FIG. 4. Compressibility of the two axes k_a and k_c for our results (k-point sampling 24x24x24 with grey diamonds, for 32x32x32 and 48x48x48 we use a fit to the results in the long dashed and solid line, respectively) and static experiments using a methanol-ethanol-water mixture (pluses, Ref. 7) and helium (average in dashed line, Ref. 12) as a pressure pressure medium.

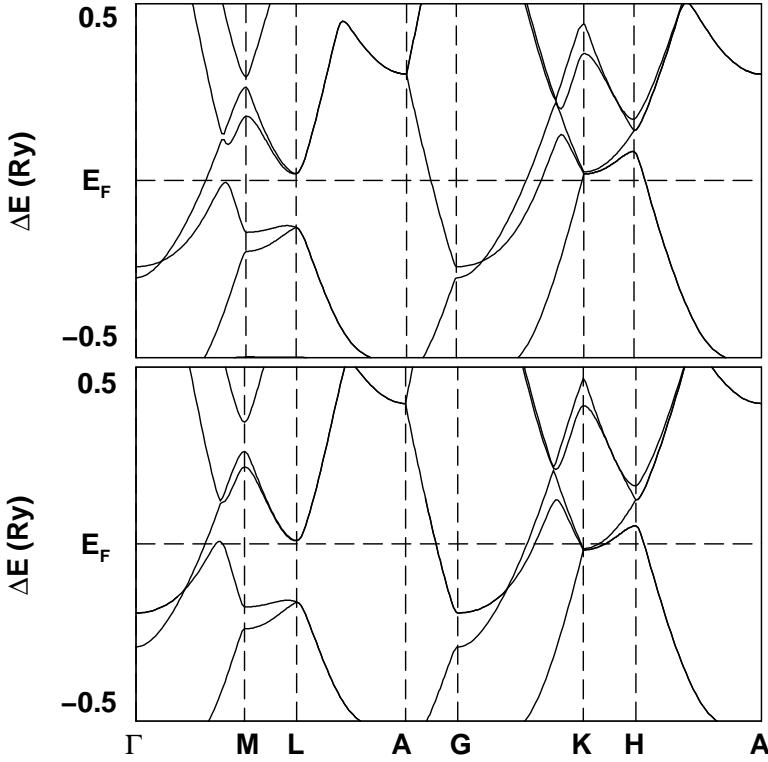


FIG. 5. Band structure for zinc along high symmetry directions in the first Brillouin zone around the Fermi energy. The upper panel shows the band structure at zero pressure ($V = 102.5 \text{ Bohr}^3$), the lower panel at $V = 95 \text{ Bohr}^3$ at their equilibrium c/a (1.91 and 1.79, respectively).

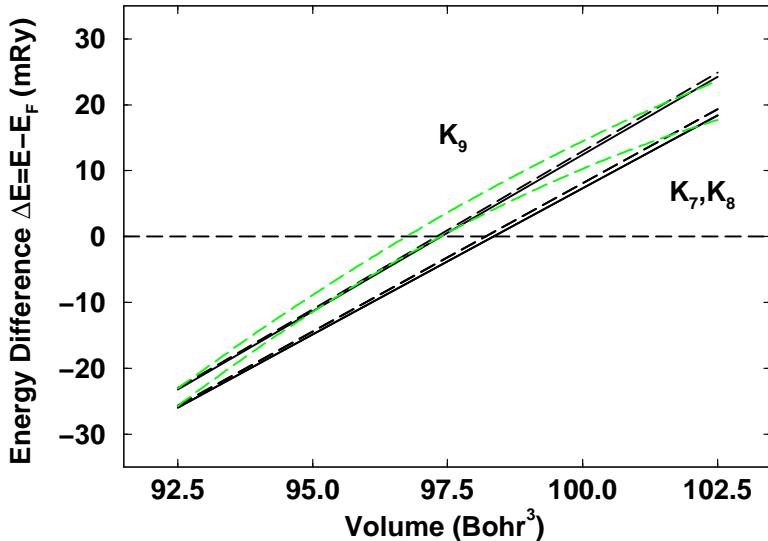


FIG. 6. Energy differences of the bands to the Fermi energy at the high symmetry point K on the first Brillouin zone boundary as a function of unit cell volume. Gray dashed, dashed, and solid lines are results for k-point sampling of 24x24x24, 32x32x32, and 48x48x48, respectively.

TABLE I. Equation of state parameters for our calculations (GGA) and experiment: V_0 , K_0 , and K'_0 are the equilibrium volume, bulk modulus, and its pressure derivative at V_0 , respectively. Due to the restricted compression range of the calculations with higher k-point sampling (32x32x32 and 48x48x48) and for the static experiments with helium as a pressure medium, we constrain K'_0 and V_0 .

method	V_0 [Bohr ³]	K_0 [GPa]	K'_0
GGA k=24	102.8	64	5.2
GGA k=32		64	
GGA k=48		63	
equilibrium properties	102.6 ²¹	60 ³³	
reduced Hugoniot ¹⁸		69	4.9
experiment MEWM ⁷		65	4.7
experiment He ¹²		61	
experiment N ⁵		63	5.2
FP-LMTO (GGA) ¹⁰	101.5	60	